ECE 445S Real-Time DSP Lab   Prof. Brian Evans

Discussion #2

Consider the discrete-time linear time-invariant (LTI) system with input $x[n]$ and output $y[n]$ given by

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

observed for $n \geq 0$.

(a) What are the initial conditions? What values should they be set to? Why?

Let $n=0$: $\underbrace{y[0]}_{\text{initial output}} = \underbrace{x[0]}_{\text{initial input}} + 2\underbrace{x[-1]}_{\text{I.C.}} + \underbrace{x[-2]}_{\text{I.C.}}$

Let $n=1$: $y[1] = x[1] + 2x[0] + x[-1]$

Let $n=2$: $y[2] = x[2] + 2x[1] + x[0]$

No additional initial conditions have appeared other than $x[-1]$ and $x[-2]$. Set the I.C.'s to zero to satisfy the necessary condition that the LTI system must be at rest at $n=0$.

(b) Find the transfer function in the z-domain, $H(z)$, of the difference equation:

Take the z-transform

$$Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

because the initial conditions are zero.

$$Y(z) = (1 + 2z^{-1} + z^{-2})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} + z^{-2} \quad \text{for } z \neq 0$$

(c) Compute the frequency response.

Because the region of convergence for $H(z)$ includes the unit circle,

$$H_{freq}(\omega) = H(z)\Big|_{z=e^{j\omega}} = 1 + 2e^{-j\omega} + e^{-j2\omega}$$

(d) Factor the frequency response in magnitude and phase form.

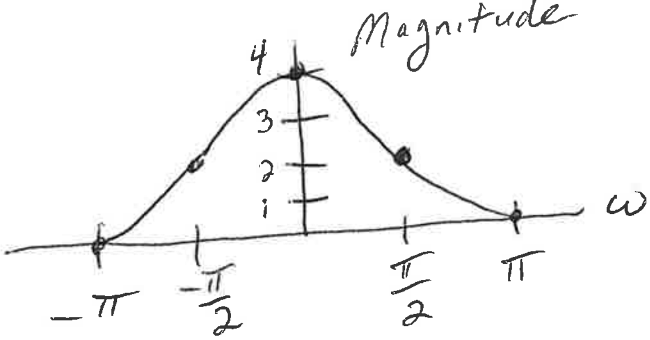
Note: This is possible for FIR filters with linear phase. Such filters have an impulse response that is even or odd symmetric about its midpoint.

Factor out the phase shift corresponding to the delay equal to the midpoint of the impulse response:

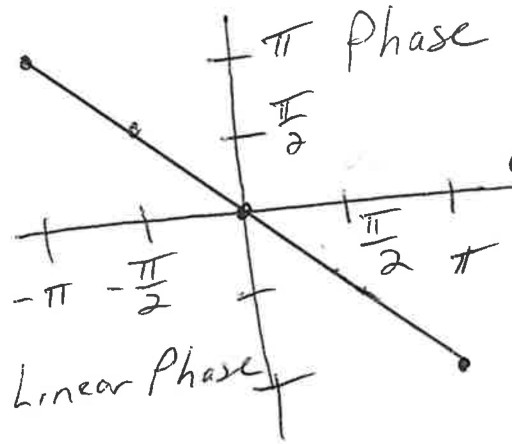
$$H_{freq}(\omega) = e^{-j\omega}\left(e^{j\omega} + 2 + e^{-j\omega}\right)$$

$$= \underbrace{e^{-j\omega}}_{\text{phase is } -\omega}\underbrace{\left(2 + 2\cos\omega\right)}_{\text{magnitude}}$$

(e) Plot the magnitude and phase responses for discrete-time frequencies from $-\pi$ to π.



Lowpass Filter



Linear Phase